How far away is the next earthquake?

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Spatial distances between subsequent earthquakes in southern California exhibit scale-free statistics, with a critical exponent $\delta \approx 0.6$, as well as finite size scaling. The statistics are independent of the threshold magnitude as long as the catalog is complete, but depend strongly on the temporal ordering of events, rather than the geometry of the spatial epicenter distribution. Nevertheless, the spatial distance and waiting time between subsequent earthquakes are uncorrelated with each other. These observations contradict the theory of aftershock zone scaling with main shock magnitude.

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What do we know about earthquakes? Either a great deal or a meager amount, depending on the point of view and on the adopted definition of an earthquake. If an earthquake is defined to be the slip on a fault (or several faults) that produces the observed seismic wave field, then we have a good understanding of earthquakes [1]. In contrast to earthquake kinematics, no satisfactory understanding exists of the physical processes in the lithosphere that cause slip on faults and are, thus, responsible for the *dynamics* of earthquakes [2]. This dynamics entangles a vast range of space and time scales and manifests itself in a number of generic, empirical features of earthquake occurrence including spatio-temporal clustering, fault traces and epicenter locations with fractal statistics, as well as the Omori and Gutenberg-Richter (GR) laws (see Refs. [3] for a review). The Omori law states that the rate of aftershocks after a main shock decays hyperbolically in time [4], while the GR law states that the size (measured in terms of the seismic moment M) distribution of earthquakes is scale-free [5].

The presence of vastly different scales has to be taken into account in order to interpret measurements correctly. For instance, to unambiguously measure the length of many natural objects, like the 'fractal' coastline of Norway, one has to specify the length of the ruler used [6, 7]. Here we ask a similar question for distances between earthquakes, and find a similar result. That is, in order to determine unambiguously the statistics for how far away the next earthquake will be, one has to specify the size of the region under consideration.

We focus on the distribution of spatial distances between the epicenters of successive earthquakes in southern California. In the past, different possibilities have been proposed including power-law behavior [8] and q-exponential (cumulative) distributions [9]. However, none of the previous studies has systematically taken into account the physical extent of the region considered, nor the threshold magnitude for including events in the analysis. Both of these quantities have recently been revealed to be crucial for capturing robust, statistical features of waiting times between subsequent earthquakes [10, 11, 12].

Here, we show that the distribution of spatial distances between successive earthquakes, larger than a threshold magnitude m, occurring within a given region of area L^2 exhibits: (1) power law behavior with an exponent $\delta \approx 0.6$, (2) finite size scaling as a function of L and (3) no dependence on the threshold magnitude m, as long as the data set is complete for that threshold. Our results also provide clear evidence that this behavior is not due to a random process bound to the geometrical structure of the collection of epicenters, but reflects the complex spatio-temporal organization of seismicity. We further argue that the exponent δ encodes information about this complex dynamics, which appears unrelated to other known properties of seismicity. Hence δ may be a new, independent exponent characterizing seismicity.

To analyze the distribution of spatial distances between successive earthquakes or "jumps", we adopt a method proposed by Bak *et al.* [10] and take the perspective of statistical physics: Neglecting any classification of earthquakes as main shocks, foreshocks or aftershocks, analyze seismicity patterns irrespective of tectonic features and place all events on the same footing. Consider spatial areas and their subdivision into square cells of length L. For each of these cells, only events above a threshold magnitude m are included in the analysis. In this way, we obtain a list of the spatial distances $\Delta r_i = |\mathbf{r}_{i+1} - \mathbf{r}_i|$ between successive events with *epicenters* $\mathbf{r}_i$ *and* $\mathbf{r}_{i+1}$ *both in the same cell of linear extent* L. Concatenating the lists of jumps obtained from each cell, a probability density function of the jumps $P_{m,L}(\Delta r)$ can be measured [13]. Since both the threshold magnitude and the length scale of the cell are arbitrary, we look for robust or universal features of this distribution that may appear when these parameters are varied.

For the SCEDC sub-catalog from southern California we study here, the reporting of earthquakes is assumed

to be homogeneous from January 1984 to December 2000 and complete for events larger than magnitude $m_c = 2.4$ [14]. Considering epicenters located within the rectangle $(120.5°W, 115.0°W) \times (32.5°N, 36.0°N)$ gives $N = 23374$ events with magnitude greater than or equal to m_c. The GR law for the cumulative distribution of earthquakes larger than magnitude m is $P_>(m) \sim 10^{-bm}$ where the seismic moment $M \propto 10^{1.5m}$, and $b = 0.95$ for this collection of events.

For $m \geq m_c$, we find that the jump distribution is described by the finite size scaling (FSS) ansatz

$$P_{m,L}(\Delta r) = \frac{f(\Delta r/L)}{L} \quad , \tag{1}$$

where the scaling function $f(x)$ decays as $x^{-\delta}$ with $\delta \approx 0.6$ for $x < 0.5$ as shown in Fig. 1. For $x > 0.5$, it decays extremely rapidly since the finite cell size requires that $f(x) = 0$ for $x > \sqrt{2}$. These observed results have far reaching implications.

First, for any L the cutoff sets in at $r \approx L/2$ and, hence, scales trivially with L. The appearance of FSS precludes the existence of any other length scale over the range where FSS holds. Thus no physical length scale exists in the range from 20km to $\approx$ 500km, in contrast to the theory of aftershock zones [15]. According to this theory, main shocks generate aftershocks within finite aftershock zones, whose extent is comparable to the rupture length $l_r = 0.02 \times 10^{0.5\,m}$km of the main event [15]. This implies that the distance between subsequent aftershocks would be limited to the size of the largest aftershock zone, which is less than 90km for the catalog analyzed here. As in Ref. [16], we find no evidence for this length scale.

Further, as δ is unambiguously less than one, the distribution $P_{m,L}(\Delta r)$ becomes non-normalizable for large L. Extrapolating our results, the finite size of the earth may play an important role in the definition of distances between subsequent earthquakes. Finally, $P_{m,L}(\Delta r)$ does not significantly depend on m for $m > m_c$, though the number of included earthquakes varies considerably with magnitude.

Although the distribution of jumps reflects a dynamical property of seismicity, the particular form of $P_{m,L}(\Delta r)$ could be determined by the geometrical structure of the collection of epicenters. A simple test can be made by randomly rearranging or 'shuffling' the temporal sequence of events, while holding the magnitude of the events and their epicenters fixed. The distribution of distances between subsequent events in the shuffled catalog is very different from the original one, as shown in Fig. 2. For the shuffled catalog, the distribution of jumps does not decay with an exponent δ, but it actually increases as a power law with exponent $\delta_{shuf} \simeq -0.14$. This can be understood from the fact that the randomly rearranged ordering gives an estimate of the distribution of distances between any two earthquakes within the

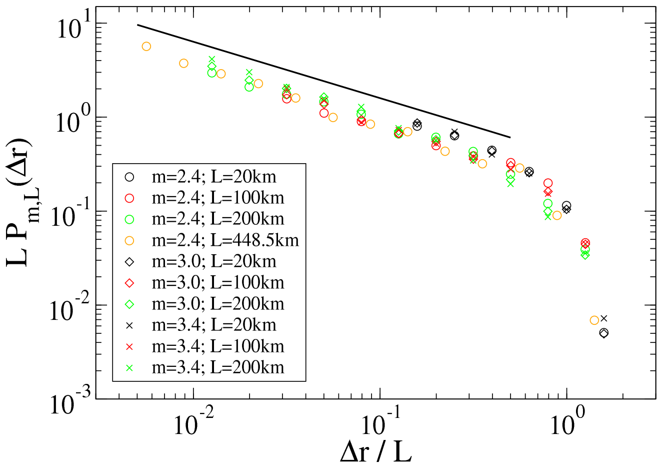

Figure 1: (Color online) Rescaled distribution of jumps, $P_{m,L}(\Delta r)$, for different values of m and L. $L = 448.5$km corresponds to the full area where there is just one cell. Values of $\Delta r <$ 2km have been discarded due to uncertainties in estimating the epicenters' locations. The solid line with exponent $\delta = 0.6$ is shown as a guide to the eye.

same cell, which is also shown in Fig. 2. The critical exponent of the cumulative distribution is, by definition, a measure of the correlation dimension D_2 of the epicenter distribution [17]. Our findings imply a fractal dimension $D_2 = 1 - \delta_{shuf} \approx 1.14$, which agrees, within statistical error, with the value obtained in [12].

The comparison between the distribution of jumps and the distribution of distances between epicenter pairs clearly proves the dynamical origin of a non-trivial δ. In particular, this exponent is not simply related to the correlation dimension. Yet, D_2 is also independent of the threshold magnitude of the earthquakes considered and the size of the region studied [18]. The latter fact and the appearance of FSS in all distributions of Fig. 2, are inconsistent with the existence of any preferred length scale, other than the cell size chosen by the observer.

The "propagation" of seismic activity is not only described by spatial distances Δr_i but also by the waiting time Δt_i between successive earthquakes i and $i+1$. Although the statistics of the waiting times has been studied recently [10, 11, 12], the propagation itself has not been analyzed. Here we consider the velocities, $v_i = \Delta r_i / \Delta t_i$, between subsequent events in each cell for events with magnitude above a threshold m and combine them, as before, into a probability density function $P_{m,L}(v)$. (Note that $P_{m,L}(v)$ is not directly related to the controversial and debated subject of aftershock diffusion, which refers to the expansion or migration of aftershock zones with time. See Ref. [19] for a review.)

Figure 3 shows that $P_{m,L}(v) \sim v^{-\eta}$ with $\eta \approx 1.0$ for intermediate v. The cutoff at large v is determined solely by L, due to the fixed temporal resolution of $\Delta t > 60$ sec implying $v_{max} = \sqrt{2} \cdot 100\text{km} \cdot 60\text{h}^{-1} \approx 8485$km/h. The

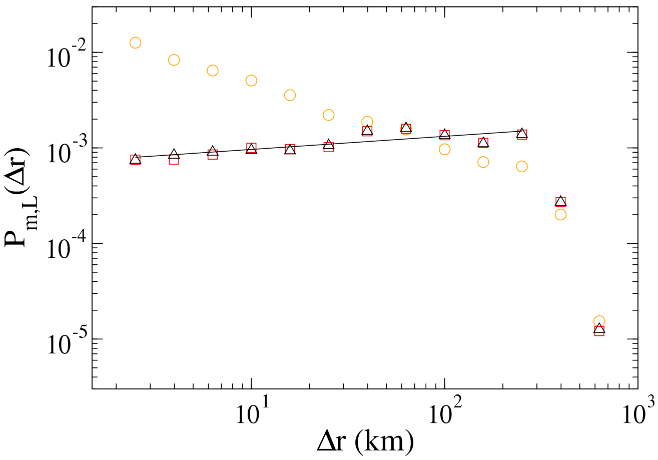

Figure 2: (Color online) The jump distribution, $P_{m,L}(\Delta r)$, for $m = 2.4$ and $L = 448.5$km. (Orange) circles correspond to the distance between successive earthquakes $\Delta r_i = |\mathbf{r}_{i+1} - \mathbf{r}_i|$ as in Fig. 1. (Red) squares correspond to $\Delta r_i = |\mathbf{r}_{\sigma(i+1)} - \mathbf{r}_{\sigma(i)}|$ where σ is a randomly chosen permutation of the integers $0 < i < N$, giving a shuffling of the catalog as described in the text. The (black) triangles correspond to the distance between *any two* earthquakes in the same cell. The solid line is a fit to the latter distribution from 2km to 200km with a critical exponent of $\delta_{shuf} = -0.14 \pm 0.03$.

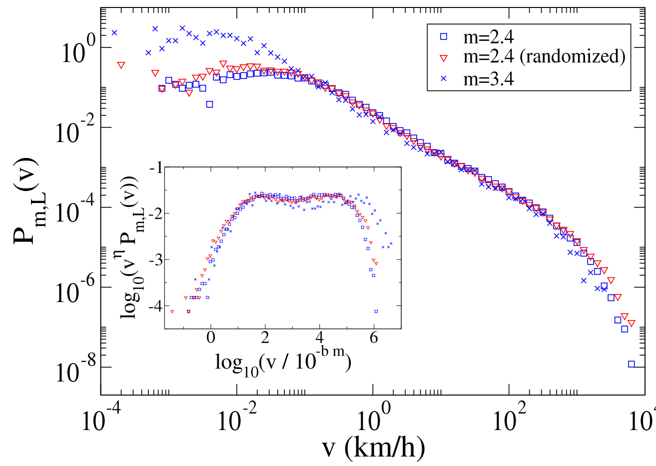

Figure 3: (Color online) The velocity distribution, $P_{m,L}(v)$, for $L = 100$km and different values of m: (Blue) crosses and boxes correspond to the original data with $v_i = \Delta r_i/\Delta t_i$. (Red) triangles correspond to a randomization $v_i = \Delta r_{\sigma(i)}/\Delta t_i$, where σ is a randomly chosen permutation of the integers $0 < i < N$. Note that $\Delta r_i > 2$km and $\Delta t_i > 60$s for all i to avoid any bias due to the uncertainties in the estimates. Inset: Rescaled distributions, using the GR exponent $b = 0.95$ and $\eta = 1.0$.

cutoff at small v depends on m. The inset of Fig. 3 shows that its position scales as $10^{-b\,m}$. If we ignore, for the moment, the variation of distances Δr, then $P_{m,L}(v) \rightarrow P_{m,L}(1/\Delta t)$. The latter distribution is obtained from the distribution of waiting times $P_{m,L}(\Delta t)$ in [10, 11, 12]. Since $P_{m,L}(1/\Delta t)\, d(1/\Delta t) = P_{m,L}(\Delta t)\, d\Delta t$, in this approximation the exponent $\eta = 2-\alpha$, where $\alpha \approx 1$ is the exponent characterizing the distribution of waiting times for intermediate arguments. Furthermore, the cutoff at small v would be controlled by the cutoff at large t in the waiting time distribution where the behavior crosses over from a power-law regime with exponent $\alpha \approx 1$ to a faster decay at $t_{cutoff} \sim 10^{b\,m}$ [10, 11, 12].

If the statistics of the waiting times and the jumps are independent, then $P_{m,L}(v)$ will only reflect the statistics of the waiting times over a range of velocities. This is due to the fact that the distribution of waiting times is much broader (approximately seven orders of magnitude) than the distribution of spatial distance (approximately three orders of magnitude) for our data. Indeed, the particular combination of spatial and temporal distances between successive earthquakes is largely random. As shown in Fig. 3, the estimate of $P_{m,L}(v)$ does not change significantly if v_i is given by $v_i = \Delta r_{\sigma(i)}/\Delta t_i$ where σ is a random permutation of the integers $0 < i < N$. This clearly proves that waiting times and jumps are independent of each other. Additionally, using the same values of m and L as in Fig. 3, the spatial and temporal distances between successive earthquakes are almost uncorrelated as measured by the cross correlation

$$r \equiv \frac{\langle (\Delta r_i - \langle \Delta r_i \rangle_i) \cdot (\Delta t_i - \langle \Delta t_i \rangle_i) \rangle_i}{\sqrt{\langle (\Delta r_i - \langle \Delta r_i \rangle_i)^2 \rangle_i} \sqrt{\langle (\Delta t_i - \langle \Delta t_i \rangle_i)^2 \rangle_i}} \approx 0.07 \quad ,$$

where $\langle \ldots \rangle_i$ denotes an average over events i.

Considering the observation that the critical exponent δ depends on the temporal order of events and is not determined solely by the geometry of the set of epicenter locations, together with the observation that the waiting times and jumps are largely uncorrelated and apparently independent of each other suggests that the exponent δ may be a new, independent exponent characterizing seismicity.

The lack of correlation between waiting times and jumps could be interpreted as an indication that the aftershock decay rate at all distances is the same, and could therefore have implications both for models of aftershock diffusion and for the rate and state/static stress friction model of aftershock triggering [19]. However, our analysis is not based on the distinction between main shocks or aftershocks. This distinction is relative [16, 20]. In fact, there is no unique operational way to distinguish between aftershocks and main shocks [10] and they are not caused by different relaxation mechanisms [21, 22]. Besides, such a classification may not always be the most useful way to describe the dynamics of seismicity. One could nevertheless consider the possibility that the scaling region in Fig. 1 can be attributed entirely to aftershocks. This would require that aftershock sequences dominate the statistics during the period and magnitude range considered. According to the traditional classification and

using aftershock zone scaling with main magnitude, the maximum distance between aftershocks would be determined by the largest events, namely the $m = 7.3$ Landers earthquake and the $m = 7.1$ Hector mine earthquake. Thus, this distance should be less than 90km. However, we find no break or change in scaling behavior for larger distances extending all the way up to the size of the region consider, of the order of 500km.

Our results are strikingly different from earlier results by Ito [8] and Abe and Suzuki [9] for California, who examined similar catalogues over a similar time span but did not take into account the length scale of observation. The latter authors also included earthquakes with magnitude as low as 0.0, and (as in [8]) found a very different jump distribution, implying that their analysis suffered from the incompleteness of the earthquake catalog at small magnitudes. Although the distribution of jumps is independent of the threshold magnitude as long as the catalog is complete, the marked difference between our results and previous ones shows that the completeness of earthquake catalogs is crucial for obtaining robust and accurate results.

As pointed out by Corral [11, 23], the (long-term) measurement of the distribution of waiting times between subsequent earthquakes — as in Ref. [10, 12] — involves an average over regions and time spans with widely different rates of seismic activity. If, instead, statistics are measured in regions and time spans with a stationary rate of earthquakes, a different distribution is obtained. A similar situation occurs for our measurement. The universal law we find for spatial distances between subsequent events holds for data sets where the rate of earthquakes is heterogeneous, i.e., for rather long time spans. Analysis at short time scales in the stationary regime, as described in Ref. [11] will be explored in a future work.

To summarize, we have shown that the distribution of spatial distances between successive earthquakes in southern California exhibits finite size scaling with a nontrivial power law exponent $\delta \approx 0.6$. Thus, in order to specify the statistics of distances between earthquakes, one has to define the length scale of observation, since no intrinsic length scale exists. This observation is in sharp contrast to the theory of aftershock zone scaling with main shock magnitude [15]. The exponent δ has a dynamical origin, but the distances and waiting times between subsequent events are found to be independent of each other. This implies that the complex dynamics of seismicity has a self-similar hierarchical structure in space and time, consistent with the hypothesis that it is a self-organized critical phenomenon [24]. Our findings can be used as benchmark tests for models of seismicity.

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